Spinor Field Realizations of $W_{2,6}$ String and W_6 String

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Abstract

In this paper the spinor field BRST charges of the W2,6 string and W6 string are constructed, where the BRST charges are graded.

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I Introduction

As is well known, much work on the W algebra and W string has received a considerable attention since 1990's[1-6]. The $BRST$ method is the simplest way to build a W string by far. In the work[7], we mentioned that the explicit forms of the $BRST$ charge for the W_4 and W_5 algebra[8,9] are so complex and it is difficult to be generalized to a general W_N string. And we point out that the reason why are in two parts: because the method is not graded and it just only belongs to the usual scalar field realizations. At the same time, we found the methods to construct the spinor field realizations of $W_{2,s}$ strings and W_N strings. We assume the $BRST$ charges of the $W_{2,s}$ strings or W_N strings are graded, this Ansatz makes their $BRST$ charges become very easy. And the exact constructions of $W_{2,5}$ string and W_5 string have been obtained[7].

Naturally the computational complexity rises rapidly when we construct the spinor field realization of $W_{2,s}$ and W_N strings with higher 's' or 'N'. It is necessary to give out a general program since the calculation by hand becomes more difficult. In fact, we have written the program which can give out the precise solutions in order to check our Ansatz and compute any solutions of the spinor realizations of $W_{2,s}$ and W_N strings. The spinor field $BRST$ charges of $W_{2,6}$ string and W_6 string have been constructed firstly by using it. Especially we improved the results of $W_{2,4}$ string in the work[10] and obtained a more general result. Of course, we also checked the results of $W_{2,3}$ and $W_{2,5}$ string[7,11], and we only spend several minutes on the calculation of $W_{2,3}$ string in particular. So the Ansatz has been showed again and all the spinor realizations of $W_{2,s}$ and W_N strings can be obtained theoretically by using our program.

This paper is organized as follows. First we give a review of the grading $BRST$ method to construct spinor field realizations of the $W_{2,s}$ strings and W_N strings. Then we introduce the thought of our program which can construct the $BRST$ charge of $W_{2,s}$ strings. Subsequently, we mainly give out the spinor field $BRST$ charges of $W_{2,6}$ string and W_6 string. And the new result of $W_{2,4}$ string is also included. Finally, a brief conclusion is given.

II Review of BRST Method about Spinor $W_{2,S}$ Strings and W_N Strings

Following Ref. [11], the $BRST$ charge for the spinor field realization of $W_{2,s}$ strings takes the form:

$$Q_B = Q_0 + Q_1, \tag{1}$$

$$Q_0 = \oint dz\ c(T^{eff} + T_\psi + KT_{bc} + yT_{\beta\gamma}), \tag{2}$$

$$Q_1 = \oint dz\ \gamma F(\psi, \beta, \gamma), \tag{3}$$

$$T_\psi = -\frac{1}{2}\partial\psi\psi, \tag{4}$$

$$T_{\beta\gamma} = s\beta\partial\gamma + (s-1)\partial\beta\gamma, \tag{5}$$

$$T_{bc} = 2b\partial c + \partial bc, \tag{6}$$

$$T^{eff} = -\frac{1}{2}\eta_{\mu\nu}\partial Y^\mu Y^\nu, \tag{7}$$

where K, y are pending constants and Y^μ is a multi-spinor. It is easy to verify that the condition $Q_0^2 = 0$ is satisfied for any 's', so the nilpotency condition $Q_B^2 = 0$ can be translated into $Q_1^2 = \{Q_0, Q_1\} = 0$. Using these conditions, the precise form of $F(\psi, \beta, \gamma)$ and exact y can be determined.

We can assume the $BRST$ charge of a general W_N string is also graded [10], and it can be given as follows:

$$Q_B = Q_0 + Q_1 + \cdots + Q_{N-2} = Q_0 + \sum_{i=1}^{N-2} Q_i. \tag{8}$$

Then the nilpotency condition $Q_B^2 = 0$ translates into

$$Q_0^2 = Q_i^2 = \{Q_0, Q_i\} = 0, \tag{9}$$

$$\{Q_i, Q_j\}_{i<j} = 0. \tag{10}$$

Observing these conditions, we can consider the equations $Q_0^2 = Q_i^2 = \{Q_0, Q_i\} = 0$ correspond to the case of $W_{2,(i+2)}$ strings for each 'i', so Q_i should take the case of $W_{2,(i+2)}$ strings directly. And the correct solution can be selected out by using the equations $\{Q_i, Q_j\}_{i<j} = 0$.

The $BRST$ charge of a spinor W_N string can be written as follows:

$$Q_B = Q_0 + Q_1 + \cdots + Q_{N-2}, \tag{11}$$

$$Q_0 = \oint dz\ c(T^{eff} + T_\psi + KT_{bc} + \sum_{i=1}^{N-2} y_i T_{\beta_i\gamma_i}), \tag{12}$$

$$Q_i = \oint dz\ \gamma_i F_i(\psi, \beta_i, \gamma_i), \tag{13}$$

$$T_\psi = -\frac{1}{2}\partial\psi\psi, \tag{14}$$

$$T_{bc} = 2b\partial c + \partial bc, \tag{15}$$

$$T_{\beta_i\gamma_i} = (i+2)\beta_i\partial\gamma_i + (i+1)\partial\beta_i\gamma_i, \tag{16}$$

$$T^{eff} = -\frac{1}{2}\eta_{\mu\nu}\partial Y^\mu Y^\nu, \tag{17}$$

where the ghost fields b, c, β_i, γ_i are all bosonic and communicating whilst the spinor field ψ has spin $\frac{1}{2}$ and is anti-communicating. They satisfy $OPEs$:

$$b(z)c(\omega) \sim \frac{1}{z-\omega}\ ,\ \beta_i(z)\gamma_i(\omega) \sim \frac{1}{z-\omega}\ ,\ \psi(z)\psi(\omega) \sim -\frac{1}{z-\omega}, \tag{18}$$

in the other case the $OPEs$ vanish.

III Thought of the Corresponding Program

According to above theory, we should obtain the spinor field realization of $W_{2,(i+2)}$ strings firstly. So producing a general program for $W_{2,s}$ strings becomes very important. Constructing the form of $F(\psi, \beta, \gamma)$ and getting the value of 'y' are our main work. The thought of our program for determining $F(\psi, \beta, \gamma)$ and 'y' can be described as follows. Firstly, write out all

the possible terms of F with ψ, β, γ considering the spin of each term 's' and ghost number zero. Then leave out all the total differential terms in $\gamma F(\psi, \beta, \gamma)$ since their contribution to Q_1 is zero. By using the nilpotency conditions $Q_1^2 = \{Q_0, Q_1\} = 0$, we can obtain corresponding equations subsequently. Finally, the value of 'y' and all the coefficients in F would be obtained by solving these equations. Thus the form of $BRST$ charge is constructed.

Here it is not necessary to list the program, if someone need it, we would like to offer it at any time.

IV Exact Spinor Field Constructions of the $W_{2,6}$ String

Apply the grading BRST method and our program, we obtained the results of the $W_{2,6}$ string as follows.

The general form of F is:

$$\begin{aligned}
F6(\psi,\beta,\gamma) = {} & f6[1]\beta^6\gamma^6 + f6[2]\beta^5\partial\gamma\gamma^4 + f6[3]\beta^4(\partial\gamma)^2\gamma^2 + f6[4]\beta^3(\partial\gamma)^3 + f6[5]\beta^4\gamma^4\partial\psi\psi \\
& + f6[6]\beta^3\partial\gamma\gamma^2\partial\psi\psi + f6[7]\beta^2(\partial\gamma)^2\partial\psi\psi + f6[8]\beta^3\gamma^3\partial^2\beta + f6[9]\beta^2\gamma^2\partial^2\beta\partial\gamma \\
& + f6[10]\beta\gamma^2\partial^2\beta\partial\psi\psi + f6[11](\partial^2\beta)^2\gamma^2 + f6[12]\beta^4\partial^2\gamma\gamma^3 + f6[13]\beta^3\partial^2\gamma\partial\gamma\gamma \\
& + f6[14]\partial\beta\beta\partial^2\gamma\partial\gamma + f6[15]\beta^2\partial^2\gamma\gamma\partial\psi\psi + f6[16]\partial\beta\partial^2\gamma\partial\psi\psi + f6[17]\partial^2\beta\beta\partial^2\gamma\gamma \\
& + f6[18]\beta^2(\partial^2\gamma)^2 + f6[19]\beta^3\gamma^3\partial^2\psi\psi + f6[20]\beta^2\partial\gamma\gamma\partial^2\psi\psi + f6[21]\beta^2\gamma^2\partial^2\psi\partial\psi \\
& + f6[22]\beta\partial\gamma\partial^2\psi\partial\psi + f6[23]\partial^2\beta\gamma\partial^2\psi\psi + f6[24]\beta\partial^2\gamma\partial^2\psi\psi + f6[25]\beta^2\gamma^3\partial^3\beta \\
& + f6[26]\partial^3\beta\beta\partial\gamma\gamma + f6[27]\partial^3\beta\gamma\partial\psi\psi + f6[28]\partial^3\beta\partial^2\gamma + f6[29]\beta^3\partial^3\gamma\gamma^2 \\
& + f6[30]\beta^2\partial^3\gamma\partial\gamma + f6[31]\beta\partial^3\gamma\partial\psi\psi + f6[32]\partial^2\beta\partial^3\gamma + f6[33]\beta^2\gamma^2\partial^3\psi\psi \\
& + f6[34]\beta\partial\gamma\partial^3\psi\psi + f6[35]\beta\gamma\partial^3\psi\partial\psi + f6[36]\partial^3\psi\partial^2\psi + f6[37]\partial^4\beta\beta\gamma^2 \\
& + f6[38]\beta^2\partial^4\gamma\gamma + f6[39]\partial\beta\partial^4\gamma + f6[40]\beta\gamma\partial^4\psi\psi + f6[41]\partial^4\psi\partial\psi \\
& + f6[42]\partial^5\beta\gamma + f6[43]\beta\partial^5\gamma + f6[44]\partial^5\psi\psi.
\end{aligned}$$

Three constructions have been worked out as follows:

(1) $y = 0$ and

$$\begin{aligned}
& f6[5] = f6[6] = f6[7] = f6[10] = f6[15] = f6[19] = f6[20] = f6[21] = f6[33] = f6[36] \\
& = f6[41] = f6[44] = 0, \quad f6[16] = f6[22] = f6[24] = f6[34] = C, \quad f6[23] = -\tfrac{1}{2}C, \\
& f6[27] = -\tfrac{1}{6}C, \quad f6[31] = f6[35] = \tfrac{2}{3}C, \quad f6[40] = \tfrac{1}{3}C,
\end{aligned}$$

where C and other coefficients are arbitrary constants but do not vanish at the same time.

(2) $y = 1$

The forms of $f6[i] (i = 1, 2, ..., 44)$ are complex comparatively. We do not list this result here.

(3) y is an arbitrary constant and

$$\begin{aligned}
& f6[5] = f6[6] = f6[7] = f6[10] = f6[15] = f6[19] = f6[20] = f6[21] = f6[33] = f6[36] \\
& = f6[41] = f6[44] = 0, \quad f6[1] = 16(-302879179L_2 + 102793785L_3)/281288882105, \\
& f6[2] = \tfrac{231}{5} f6[1], \quad f6[3] = 2(-379632019L_2 + 102044487L_3)/66420043, \\
& f6[4] = 242(-58153384L_2 + 10140597L_3)/332100215, \\
& f6[8] = 2(2065520459L_2 - 5774235L_3)/3653102365, \\
& f6[9] = 9(168697384L_2 - 384949L_3)/66420043, \\
& f6[11] = (4761505236L_2 + 406049487L_3 - 531360344L_5)/265680172, \\
& f6[12] = 3(-730884597L_2 + 350022565L_3)/1461240946,
\end{aligned}$$

$f6[13] = 3(-1023320876L_2 + 227882885L_3)/66420043,$
$f6[14] = 3(72905938844L_2 - 1923836317L_3 - 5313603440L_5)/664200430,$
$f6[16] = L_1, \quad f6[17] = 123(-20616556L_2 + 22289L_3)/265680172,$
$f6[18] = 3(8124502060L_2 - 245230307L_3 - 531360344L_5)/265680172,$
$f6[22] = L_1, \quad f6[23] = -\frac{1}{2}L_1, \quad f6[24] = L_1, \quad f6[25] = L_2,$
$f6[26] = (-11324245500L_2 - 806175417L_3 + 1062720688L_5)/132840086, \quad f6[27] = -\frac{1}{6}L_1,$
$f6[28] = (-13043019308L_2 + 255536149L_3 + 996300645L_4 - 664200430L_6)/664200430,$
$f6[29] = L_3, \quad f6[30] = (26247460620L_2 - 728072793L_3 - 1062720688L_5)/265680172,$
$f6[31] = \frac{2}{3}L_1, \quad f6[32] = L_4, \quad f6[34] = L_1, \quad f6[35] = \frac{2}{3}L_1,$
$f6[37] = (-23291863284L_2 - 2029012023L_3 + 2656801720L_5)/1328400860,$
$f6[38] = L_5, \quad f6[39] = L_6, \quad f6[40] = \frac{1}{3}L_1,$
$f6[42] = (21371821812L_2 - 2351490636L_3 - 1660501075L_4 + 1328400860L_6)/6642004300,$
$f6[43] = 2(-858895785L_2 - 3491642L_3 + 66420043L_6)/332100215,$

where $L_1, L_2, L_3, L_4, L_5, L_6$ are arbitrary constants but do not vanish at the same time.

V Exact Spinor Field Constructions of W_6 String

Using our program we also checked the results of the $W_{2,3}$ string and the $W_{2,5}$ string, and got a more general solution of the $W_{2,4}$ string. Furthermore, we find the solutions of $W_{2,s}(s = 3, 4, 5, 6)$ strings are very standard, that is, there are three solutions for each 's'. In these solutions, the values of 'y' correspond to '0','1' and an arbitrary constant respectively. Thus $Q_N(N = s-2)$ can be either of $Q_N^{(1)}$, $Q_N^{(2)}$ and $Q_N^{(3)}$. We standardize these solutions as follows: $Q_N^{(1)}$ corresponds to the solution of $y = 0$, $Q_N^{(2)}$ to $y = 1$, and $Q_N^{(3)}$ corresponds to the case that $'y'$ is an arbitrary constant. By this assumption, we rewrite the results of $W_{2,3}$ string and give out the new constructions of $W_{2,4}$ string. The solutions of $W_{2,5}$ string are the same with that in Ref.[7].

V.1 The solutions of $W_{2,3}$ string

$$F3 = f3[1]\beta^3\gamma^3 + f3[2]\beta\gamma^2\partial\beta + f3[3]\partial\beta\partial\gamma + f3[4]\beta\gamma\psi\partial\psi + f3[5]\beta\partial^2\gamma + f3[6]\psi\partial^2\psi.$$

(1) $y = 0$ and

$f3[4] = f3[6] = 0,$

where other coefficients are arbitrary constants but do not vanish at the same time.

(2) $y = 1$ and

$f3[1] = \frac{1}{150}(-7N_1 + 3N_2), \quad f3[2] = \frac{1}{15}(7N_1 - 3N_2), \quad f3[3] = N_1,$
$f3[4] = -\frac{2}{5}(11N_1 - 39N_2), \quad f3[5] = N_2, \quad f3[6] = 11N_1 - 39N_2,$

where N_1 and N_2 are arbitrary constants but do not vanish at the same time.

(3) y is an arbitrary constant and

$f3[4] = f3[6] = 0, \quad f3[1] = -8P, \quad f3[2] = 80P, \quad f3[3] = 195P, \quad f3[5] = P,$

where P is an arbitrary nonzero constant.

V.2 The new solutions of $W_{2,4}$ string

$$F4 = f4[1]\beta^4\gamma^4 + f4[2](\partial\beta)^2\gamma^2 + f4[3]\beta^3\gamma^2\partial\gamma + f4[4]\beta^2(\partial\gamma)^2 + f4[5]\beta^2\gamma^2\psi\partial\psi$$
$$+ f4[6]\partial\beta\gamma\psi\partial\psi + f4[7]\beta\partial\gamma\psi\partial\psi + f4[8]\beta\partial^2\beta\gamma^2 + f4[9]\partial^2\beta\partial\gamma + f4[10]\partial\beta\partial^2\gamma$$

$+ f4[11]\partial\psi\partial^2\psi + f4[12]\beta\partial^3\gamma + f4[13]\psi\partial^3\psi.$

(1) $y = 0$ and

$f4[5] = f4[6] = f4[7] = f4[11] = f4[13] = 0,$

where other coefficients are arbitrary constants but do not vanish at the same time.

(2) $y = 1$ and

$f4[1] = \frac{1}{468930}(-58Z_1 - 114Z_2 + 18207Z_3), \quad f4[2] = \frac{1}{15312}(-116Z_1 - 63Z_2 + 15426Z_3),$
$f4[3] = \frac{70}{3}f4[1], \quad f4[4] = \frac{1}{232}(-5Z_2 + 114Z_3), \quad f4[5] = \frac{1}{21}(-4Z_1 + 3Z_2),$
$f4[6] = Z_1, \quad f4[7] = Z_2, \quad f4[8] = Z_3, \quad f4[9] = \frac{1}{696}(-7Z_2 + 1134Z_3 + 232Z_4),$
$f4[10] = Z_4, \quad f4[11] = \frac{1}{7}(33Z_1 + Z_2), \quad f4[12] = \frac{35}{1392}(Z_2 - 798Z_3 + 928Z_4),$
$f4[13] = \frac{1}{21}(-11Z_1 + 16Z_2),$

where Z_1, Z_2, Z_3 and Z_4 are arbitrary constants but do not vanish at the same time.

(3) y is an arbitrary constant and

$f4[5] = f4[6] = f4[7] = f4[11] = f4[13] = 0, \quad f4[1] = \frac{867}{22330}V_1, \quad f4[2] = \frac{2571}{2552}V_1,$
$f4[3] = \frac{289}{319}V_1, \quad f4[4] = \frac{57}{116}V_1, \quad f4[8] = V_1, \quad f4[9] = \frac{189}{116}V_1 + \frac{1}{3}V_2,$
$f4[10] = V_2, \quad f4[12] = -\frac{133}{232}V_1 + \frac{2}{3}V_2,$

where V_1 and V_2 are arbitrary constants but do not vanish at the same time.

Comparing these new results with that in Ref.[10], we found they are more general.

V.3 Exact spinor field constructions of W_6 string

In this case the $BRST$ charge can be written as $Q_B = Q_0 + Q_1 + Q_2 + Q_3 + Q_4$, where $Q_i (i = 1, 2, 3, 4)$ must be one of the spinor field realization of $W_{2,s} (s = 3, 4, 5, 6)$ string respectively. The selection rule equation(10) namely $\{Q_1, Q_2\} = \{Q_1, Q_3\} = \{Q_1, Q_4\} = \{Q_2, Q_3\} = \{Q_2, Q_4\} = \{Q_3, Q_4\} = 0$. For these selection rules, we list two tables in which the correct combinations are denoted as sign '$\surd$' whilst the incorrect combinations are denoted as sign '$\times$'. Then the exact constructions of the spinor field grading $BRST$ charge for the W_6 string are obtained. Especially we give out two simple correct combinations here.

V.3.1 The simplest combination of $Q_0, Q_1^{(1)}, Q_2^{(1)}, Q_3^{(1)}$ and $Q_4^{(1)}$

$$Q_B = \oint dz \; [\; c(-\frac{1}{2}\eta_{\mu\nu}\partial Y^\mu Y^\nu - \frac{1}{2}\partial\psi\psi + 2Kb\partial c + K\partial bc) + f_1\beta_1^3\gamma_1^3 + f_2\beta_2^4\gamma_2^4 + f_3\beta_3^5\gamma_3^5 + f_4\beta_4^6\gamma_4^6 \;]$$

where $f_i (i = 1, 2, 3, 4)$ are arbitrary nonzero constants.

This is one of the simplest formulae of Q_B.

V.3.2 Another simple combination of $Q_0, Q_1^{(3)}, Q_2^{(3)}, Q_3^{(3)}$ and $Q_4^{(3)}$

$$\begin{aligned} Q_B = \oint dz \; [\; & c(-\frac{1}{2}\eta_{\mu\nu}\partial Y^\mu Y^\nu - \frac{1}{2}\partial\psi\psi + 2Kb\partial c + K\partial bc + 3\beta_1\partial\gamma_1 + 2\partial\beta_1\gamma_1 \\ & + 4\beta_2\partial\gamma_2 + 3\partial\beta_2\gamma_2 + 5\beta_3\partial\gamma_3 + 4\partial\beta_3\gamma_3 + 6\beta_4\partial\gamma_4 + 5\partial\beta_4\gamma_4) \\ & + g_1(-8\beta_1^3\gamma_1^3 + 80\beta_1\gamma_1^2\partial\beta_1 + 195\partial\beta_1\partial\gamma_1 + g_1\beta_1\partial^2\gamma_1) \\ & + g_2(\partial^2\beta_2\partial\gamma_2 + 3\partial\beta_2\partial^2\gamma_2 + 2\beta\partial_2^3\gamma_2) \\ & + g_3(3\partial^2\beta_3\partial^2\gamma_3 + \partial\gamma_3\partial^{(3)}\beta_3 + 2\partial\beta_3\partial^{(3)}\gamma_3) \end{aligned}$$

$$+ g_4(6\partial^3\beta_4\partial^2\gamma_4 + 4\partial^2\beta_4\partial^3\gamma_4 - \partial^5\beta_4\gamma_4)]$$

where $g_i(i = 1, 2, 3, 4)$ are any arbitrary nonzero constants.

V.3.3 The complete combinations of $Q_i(i = 1, 2, 3, 4)$

The exact combinations could be found in Table 1 and Table 2.

Table 1: The precise combinations of Q_1,Q_2,Q_3 and Q_4

combinations	$Q_2^{(1)}$	$Q_2^{(2)}$	$Q_2^{(3)}$	$Q_3^{(1)}$	$Q_3^{(2)}$	$Q_3^{(3)}$	$Q_4^{(1)}$	$Q_4^{(2)}$	$Q_4^{(3)}$
$Q_1^{(1)}$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
$Q_1^{(2)}$	$\surd$	$\times^1$	$\surd$	$\surd$	$\times^2$	$\surd$	$\times^*$	$\times^4$	$\times^*$
$Q_1^{(3)}$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
$Q_2^{(1)}$	$-$	$-$	$-$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
$Q_2^{(2)}$	$-$	$-$	$-$	$\surd$	$\times^3$	$\surd$	$\times^*$	$\times^5$	$\times^*$
$Q_2^{(3)}$	$-$	$-$	$-$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
$Q_3^{(1)}$	$-$	$-$	$-$	$-$	$-$	$-$	$\surd$	$\surd$	$\surd$
$Q_3^{(2)}$	$-$	$-$	$-$	$-$	$-$	$-$	$\times^*$	$\times^6$	$\times^*$
$Q_3^{(3)}$	$-$	$-$	$-$	$-$	$-$	$-$	$\surd$	$\surd$	$\surd$

Table 2: The special combinations of Q_i and Q_j when $f6[31] = 0$

combinations	$Q_j^{(1)}$	$Q_j^{(2)}$	$Q_j^{(3)}$
$Q_i^{(1)}$	$\surd$	$\surd$	$\surd$
$Q_i^{(2)}$	$\surd$	$\times$	$\surd$
$Q_i^{(3)}$	$\surd$	$\surd$	$\surd$

[1]If we take $f4[6] = f4[7] = 0$(we name it condition 1) , this combination will be correct.
[2]If we take $f5[8] = f5[9] = f5[14] = 0$(we name it condition 2), this combination will be correct.
[3]If we take condition 1 or 2, this combination will be correct.
[4]If we take $f6[6] = f6[9] = f6[10] = f6[15] = f6[20] = f6[25] = f6[29] = f6[31] = 0$(we name it condition 3), this combination will be correct.
[5]If we take condition 1 or 3, this combination will be correct.
[6]If we take condition 2 or 3, this combination will be correct.
[*]If we take $f6[31] = 0$, all the '$\times^*$' in Table 1 will become '$\surd$'. Then each part of Table 1 changes to Table 2.

VI Conclusion

In conclusion, the spinor field grading $BRST$ charges of $W_{2,6}$ string and W_6 string have

been constructed. And a special program was obtained to construct spinor field $BRST$ charges of a general $W_{2,s}$ strings by using the $BRST$ method. With this procedure, the results are more accurate, and the process is accelerated rapidly. We have checked the solutions of $W_{2,3}$ string as well as $W_{2,5}$ string and given out a more general result of $W_{2,4}$ string in spinor field. Observing these solutions, we found they are standard. Of course the more realizations of $W_{2,s}$ strings can be calculated by using our program. Using these results, we will discuss their physical states in our next work.

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